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Nota interna: n° 163
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ABSTRACT.

The cross section of the kaon production reaction by high energy neutrinos, $\nu + p \rightarrow p + \ell^- + K^+$, is computed in the peripheral approximation. Numerical estimates are made with specific form factors derived from the dispersion theory. The cross section depends on such a dynamical model, hence it can serve as a criterion of the model.

1. The form factors of $K^+_{\ell 3}$ decay have been studied as a function of s, the square of the four-momentum transfer, by several authors in dispersion theoretic approach[1, 2, 3,)]. Since the range of s is rather narrowly limited in the decay reaction[4)], the behaviour of the form factors is not fully revealed in the decay process. Hopefully future experiments with high energy neutrinos will provide a means to study the form factors in a wider range of s. We point out here that the reactions of charged kaon production

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by neutrinos,

(1) $$\nu + p \rightarrow p + \ell^{-} + K^{+}$$

(2) $$\nu + n \rightarrow n + \ell^{-} + K^{+},$$

are particularly fit for such a purpose.

Consider the process of kaon production by high energy neutrinos or antineutrinos from a nucleon,

(3) $$\nu + N \rightarrow N + \ell^{-} + K$$

(4) $$\bar{\nu} + N \rightarrow N + \ell^{+} + K,$$

without specifying the charge channel. The reactions may take place in two ways, namely via production by the baryon core or by the meson cloud. If the pole diagrams are assumed to be dominant, the mechanisms shown in Fig. 1 and in Fig. 2 play the principal rôle. The core diagram of Fig. 1, however, does not exist in the reactions (1) and (2), because of charge conservation at the weak vertex and of strangeness conservation at the strong vertex respectively. The peripheral diagram of Fig. 2 is naturally associated with the leptonic decay diagram of the kaon.

The reactions of charged kaon production by antineutrinos,

(5) $$\bar{\nu} + p \rightarrow p + \ell^{+} + K^{-}$$

(6) $$\bar{\nu} + n \rightarrow n + \ell^{+} + K^{-}$$

require both diagrams, but the core diagram is supposed to be of minor importance because of the appearance of a heavy virtual particle. The peripheral diagram gives the cross sections for the reaction (5) and (6) identical to those of the reactions (1) and (2) because of the reality properties of the decay form factors, that follow from time reversal invariance.

Similarly in the K^0 production processes,

$$\nu + n \rightarrow p + \ell^{-} + K^{0}$$
$$\bar{\nu} + p \rightarrow n + \ell^{+} + K^{0},$$

the core diagram is not allowed by strangeness conservation at the strong vertex, and in the $\overline{K}^o$ production processes,

$$\nu + n \rightarrow p + \ell^- + \overline{K}^o$$
$$\overline{\nu} + p \rightarrow n + \ell^+ + K^o,$$

both diagrams are allowed. However, the peripheral diagrams for these neutral kaon production processes call for the knowledge of the leptonic decay form factors of the neutral kaon,

$$K^o \rightarrow \pi^- + \ell^+ + \nu$$
$$K^o \rightarrow \pi^+ + \ell^- + \overline{\nu} ,$$

which are less known than those of the charged kaon.

In this paper we present the cross section formula of the reaction (1) based on the peripheral mechanism and the expression for the decay rate of $K^+_{\ell 3}$ for arbitrary form factors (Section 2). By adopting the specific form factors derived from the pole approximation in dispersion theory we give numerical estimates of the cross section in view of future experiments (Section 3).

2. The threshold neutrino energy in the laboratory system of the reaction (1) is 0. 79 GeV and 0. 62 GeV for the muon and electron mode respectively. The standard manipulation of the peripheral diagram of Figure 2[5) 6)] gives the partial cross section

$$\frac{d^3\sigma}{dxdydz} = \sigma_o \cdot \frac{1}{(\xi - 1)^2} \cdot \frac{y}{(x+y)(y+\mu)^2} \cdot \Big[|f(z)|^2 \big((x-\lambda) \cdot (x+y-z-\lambda) - \kappa (x+y) \big) + 2\,\mathrm{Re}(f(z)g^*(z)) \cdot \lambda (x+y-z-\lambda) + |g(z)|^2 \cdot \lambda (z+\lambda) \Big] , \tag{7}$$

where x, y, z, are dimensionless invariant variables

$$x = \frac{-(q_1+q)^2}{m_p^2} , \qquad y = \frac{(p_2 - q_2)^2}{m_p^2} , \qquad z = \frac{(p_1 - q_1)^2}{m_p^2} ,$$

λ, μ, κ are numerical parameters of the masses

$$\lambda = \left(\frac{m_e}{m_p}\right)^2, \quad \mu = \left(\frac{m_\pi}{m_p}\right)^2, \quad \kappa = \left(\frac{m_K}{m_p}\right)^2,$$

and ξ is a dimensionless parameter which specifies the incident energy,

$$\xi = \left(\frac{W}{m_p}\right)^2,$$

W being the total energy in the over-all barycentric system related to the incident neutrino energy in the laboratory system E_ν by.

$$E_\nu = \frac{W^2 - m_p^2}{2m_p} .$$

The factor σ_0 is defined by

$$\sigma_0 = \frac{1}{\cancel{128}\,32\,\pi^2} \cdot \frac{g^2}{4\pi} \cdot G^2 m_p^4 \cdot \frac{1}{m_p^2} = \cancel{5.29}\;21.2 \times 10^{-40}\ \mathrm{cm}^2,$$

where g is the renormalized neutral pion-proton coupling constant ($g^2/4\pi \sim$ ~ 15) and G is the Fermi coupling constant ($G^2 m_p^4 \sim 1.0 \times 10^{-10}$). The functions f and g are dimensionless form factors that appear in the matrix element of the kaon-pion current

$$\langle \pi^0 / j_\mu / K^+ \rangle = \frac{1}{(2\pi)^3} \cdot \frac{1}{\sqrt{2\,\omega_K}} \cdot \frac{1}{\sqrt{2\,\omega_\pi}} \cdot \tag{8}$$
$$\left[\frac{f}{2}(p_K + p_\pi)_\mu + \left(\frac{f}{2} - g\right)(p_K - p_\pi)_\mu\right],$$

where p and ω stand for the four-momentum and energy of the indicated meson, and are functions of $s = (p_K - p_\pi)^2$. In the peripheral diagram they are functions of z by the momentum conservation at the weak vertex.

$$\frac{(p_K - p)^2}{m_p^2} = \frac{(p_1 - q_1)^2}{m_p^2} = z.$$

Specializing Eqn. (7) for the electron mode we put $\lambda = 0$ and obtain

$$(9) \qquad \frac{d^3\sigma}{dxdydz} = \sigma_0 \cdot \frac{1}{(\xi-1)^2} \cdot \frac{y\left[(x-\kappa)(x+y)-xz\right]}{(x+y)(y+\mu)^2} \left|f(z)\right|^2,$$

which enables us to measure the form factor f(z) directly as a function of z from the angular distribution of the kaon under the condition of fixed x and y, if the experimental data are copious enough to allow such a measurement.

The other partial cross sections $d^2\sigma / dxdy$, $d\sigma/dx$ and the total cross section $\sigma(\xi)$ are obtained by integrating Eqn. (7) with respect to z, y, x in succession between the limits

$$(10) \quad z_{\substack{max\\min}}(x,y) = -\lambda + \frac{(x+y)(x-\kappa+\lambda)}{2x} \pm \frac{x+y}{2x}\sqrt{x^2-2x(\kappa+\lambda)+(\kappa-\lambda)^2}$$

$$(11) \quad y_{\substack{max\\min}}(x) = \frac{1}{2\xi}\left[(\xi-1)^2 - (\xi+1)x \pm (\xi-1)\sqrt{x^2-2x(\xi+1)+(\xi-1)^2}\right]$$

$$(12) \quad x_{max}(\xi) = (\sqrt{\xi}-1)^2, \qquad x_{min} = (\sqrt{\kappa}+\sqrt{\lambda})^2.$$

With the form factors defined in Eqn. (8) the $K^+_{\ell 3}$ decay rate becomes

$$(13) \quad \begin{aligned} w(K_{\ell 3}) = w_0 \int_{\beta}^{(1-\sqrt{\alpha})^2} & \sqrt{\zeta^2 - 2\zeta(1+\alpha) + (1-\alpha)^2} \; \left(1-\frac{\beta}{\zeta}\right)^2 \times \\ \times & \left[\left|f(\zeta)\right|^2 \left\{\frac{1}{6}\left(\zeta^2 - 2\zeta(1+\alpha) + (1-\alpha)^2\right) + \beta\right\}\left(1 - \frac{2\beta}{\zeta}\right) - \right. \\ & \left. - \mathrm{Re}(f(\zeta)\, g^*(\zeta))\, \beta(\zeta + 1 - \alpha) + \left|g(\zeta)\right|^2 . \beta\zeta \right] d\zeta , \end{aligned}$$

where α , β are numerical parameters

$$\alpha = \left(\frac{m_\pi}{m_K}\right)^2, \qquad \beta = \left(\frac{m_\ell}{m_K}\right)^2,$$

w_o is given by

$$w_o = \frac{G^2 m_K^5}{128\,\pi^3} = 1.48 \times 10^9 \quad \text{sec}^{-1},$$

and the argument of the form factors is related to the integration variable ζ by

$$\frac{(p_K - p_\pi)^2}{m_K^2} = -\zeta .$$

3. In general the kaon-pion current consists of two currents with definite isospin character $I = 3/2$ and $I = 1/2$. For each component current a set of form factors f and g are defined by Eqn. (8), and they are assumed to satisfy certain dispersion relations. If all the intermediate states but the one kaon plus one pion state are neglected, the dispersion relations permit an approximate solution of the form factors in terms of the s - and p - wave phase shifts of the kaon-pion scattering in that particular isospin channel[1-3]. The form factors take a particularly simple form if the kaon plus pion state is dominated by a resonance (pole approximation). Such a resonance state may be associated with the experimentally observed K^* particle at 885 MeV[7]. The K^* particle has $I = 1/2$, but its intrinsic angular momentum J is not yet identified; actually two appearently contradicting experiments suggest $J = 0$[8] or $J = 1$[7),9),10].

For the sake of definiteness let us assume the $/\Delta I/ = 1/2$ selection rule, thus confining the kaon-pion current in $I = 1/2$ component only, and the K^* resonance either in $J = 0$ or 1 state. The form factors then become[3),11].

i) $J = 0$ (s-wave resonance)[12]

(14) $$f(z) = C_o, \quad g(z) = \frac{C_o}{z}\left(1 - \frac{K - \mu}{z + p}\right),$$

ii) $J = I$ (p-wave resonance)[13]

(15) $$f(z) = C_1 \frac{\rho}{z + \rho}, \quad g(z) = \frac{C_1}{2} \cdot \frac{\rho + \kappa - \mu}{z + \rho}$$

where

$$\rho = \left(\frac{m_K^*}{m_p}\right)^2,$$

and C_o and C_1 are constants which may be determined by the experimental decay rate.

Eqn. (13) is analytically integrable for the electron mode with the form factors (14) and (15). The decay rate reads

$$(16) \qquad w(K_{e3}) = w_o C_o^2 \times 2.38 \times 10^{-2}$$

$$(17) \qquad w(K_{e3}) = w_o C_1^2 \times 2.65 \times 10^{-2}$$

for the s - and p - wave resonance respectively. Matching these expressions to the experimental K_{e3} decay rate[14)]

$$w(K_{e3})_{exp} = 4.09 \times 10^6 \quad sec^{-1}$$

we find

$$C_o^2 = 0.117, \qquad G^2 = 0.105$$

Numerical integration of Eqn. (13) with the form factors (14) and (15)yields the $K_{\mu 3}$ decay rate

$$(18) \qquad w(K_{\mu 3}) = w_o c_o^2 \times 1.66 \times 10^{-2}$$

$$(19) \qquad w(K_{\mu 3}) = w_o c_1^2 \times 1.74 \times 10^{-2}$$

for the s - and p-wave resonance respectively, and the comparison with the experimental $K_{\mu 3}$ decay rate[14)]

$$w(K_{\mu 3})_{exp} = 3.92 \times 10^6 \; sec^{-1}$$

gives

$$C_o^2 = 0.160, \qquad C_1^2 = 0.153.$$

As an average the following estimate is adopted for C_o and C_1:

$$(20) \qquad C_o^2 = 0.14 \quad or \quad C_o = 0.37$$

$$(21) \qquad C_1^2 = 0.13 \quad or \quad C_1 = 0.36.$$

Numerical integration of the Eqn. (7) with Eqns. (10), (11), (12), (14), (15),(20), (21) yields the total cross sections $\sigma(\xi)$ summarized in Table 1 and 2. For the purpose of comparison the total cross sections of the "elastic" process

$$\bar{\nu} + n \rightarrow p + e^-$$

and the pion production process

$$\nu + p \rightarrow p + e^- + {}^+$$

at the corresponding energies are quoted from the literature[15) 16)] and compiled in Table 3.

We observe that at moderate incident energies sufficiently above the threshold there is a difference of a factor 2 $\sim$ 10 in the total cross section for two choices of the form factors. With the approximation of vanishing lepton mass the momentum transfer variable is

$$z = \frac{(p_1 - q_1)^2}{m_p^2} = \frac{2E_\nu E_\ell}{m_p^2} \quad (1 - \cos\theta),$$

where E_ℓ and θ are the energy and production angle of the lepton in the laboratory system respectively. In high energy region the incident momentum will be carried away mainly by leptons in the forward direction, therefore as a kinematical upper bound z ranges from 0 to z max,

$$z_{max} \sim \left(\frac{E_\nu}{m_p}\right)^2 \quad \theta^2_{max},$$

and the range of z increases rapidly with increasing incident energy, thus making the behaviour of the form factors more marked. For the p-wave resonance Eqn. (15), $/f(z)/^2$ and $/g(z)/^2$ fall off to their maximum value at z=o abready at $z \sim 2$. It is clear that the larger cross section predicted by the s- wave resonance form factors comes from the constancy of the function f, while the p-wave resonance form factors behave essentially as cut-off factors.

From Eqns. (16), (17), (18), (19) the branching ratio of the muon to electron decay mode reads

$$\frac{w\ (K_{\mu 3})}{w\ (K_{e3})} = 0.70 \text{ and } 0.66$$

for the s - and p-wave resonance respectively[11]. These values are hardly in good agreement with the experimental finding of 1.0 ± 0.2, and naturally our cross section in Table 1 and 2 must be regarded only as order of magnitude estimates.

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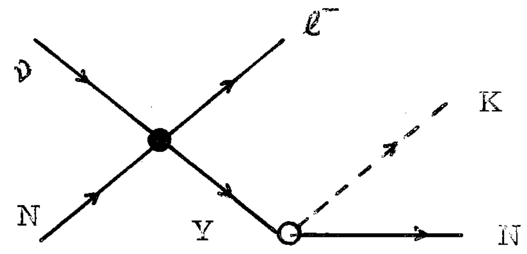

Fig. 1 - The pole diagram for the production in the baryon core (core diagram). The white circle represents the strong interaction and the black the weak interaction.

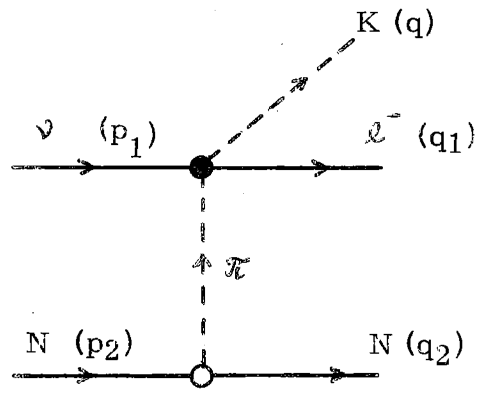

Fig. 2 - The pole diagram for the production in the meson cloud (peripheral diagram). The quantities inside the bracket stand for the four-momentum of the particle.

E_ν (GeV	ξ	$\sigma(\nu+p \to p+\mu^-+K^+)$	$\sigma(\nu+p\to p+e^-+K^+)$
1	3.13	4.2×10^{-3}	9.6×10^{-3}
2	5.26	0.207	0.292
3	7.40	0.723	0.978
5	11.7	2.46	3.29
10	22.3	9.55	12.7

Table 1. Total cross section of the reaction (1) in units of 10^{-40} cm^2 for the s-wave K^*resonance Eqn. (14).

E_ν (GeV)	$\sigma(\nu+p\to p+\mu^-+K^+)$	$\sigma(\nu+p\to p+e^-+K^+)$
1	3.1×10^{-3}	5.4×10^{-3}
2	0.095	0.103
3	0.241	0.252
5	0.542	0.554
10	1.14	1.15

Table 2. Total cross section of the reaction (1) in units of 10^{-40} cm^2 for the p-wave K^*resonance Eqn. (15).

E_ν (GeV)	$\sigma(\nu+n\to p+e^-)$	$\sigma(\nu+p\to p+e^-+\pi^+)$
1	88.6	1.07
2	83.6	5.16
3	81.0	10.7
5	78.5	20.8
10	76.7	40.

Table 3. Total cross section of the elastic and pion production reactions in units of 10^{-40} cm^2.

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